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Proyecto CLIMA

Behavioral Decision Theory:

How Judgments and Decisions are Made Under Uncertainty

Lesson 2

Normative and Prescriptive Decision Models

Types of Models

- Normative
  - A standard that defines "best" way of achieving some goal
    - Goals include maximization, optimization, consistency across situations and contexts
- Descriptive
  - Describe how people normally think and decide
  - Often expressed in terms of heuristics/simple rules
  - Can also describe regularities of behavior mathematically
- Prescriptive
  - Prescribe how we "ought" to think or act
  - Often derive from normative models, but can also involve heuristics/shortcuts
  - Successful prescriptive models will also incorporate lessons from descriptive models

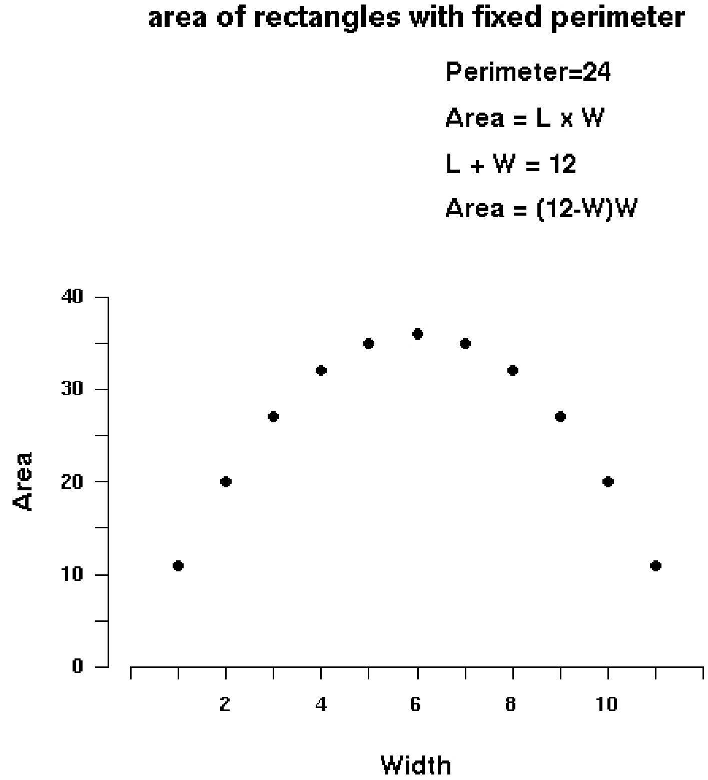
Decision Making as Constrained Optimization

- Specification of Objective Function
  - Objective function specifies ***decision rule***

- Identification of Constraints
  - Physical (engineering) models have physical constraints
  - Normative decision models have logic and consistency constraints (axioms)
  - Descriptive decision models have cognitive and affective constraints

Example of a physical constraint optimization problem

- Maximize the rectangular area that can be enclosed by 24 feet of fencing material



"A Decision Theorist Reads the Newspaper"

- New York Times, Sept. 3, 2006 story on air traffic controller staffing decisions made by Federal Aviation Agency and airports
  - Conflicting goals
    - Maximize public safety
    - Minimize expenses
  - Optimization involves specification of a tradeoff factor
    - Relative importance of safety vs. costs
      - Adjustments of relative importance weights with feedback
        - e.g., accidents that involve loss of lives, like the crash of a Boing737 of the Brazilian airline Gol Transportes Aereos, on September 29, 2006, on route from Manaus to Brasilia, causing 154 fatalities, no survivors (after mid-air collision with business jet which landed safely with some damage to the aircraft)

Where do decision rules come from?

- **They are *learned***
  - by experience (induction)
    - “learning by getting hurt”
  - by observing others
    - “learning by watching”
  - by explicit instruction
    - “learning by being told”

- **They are *deduced***
  - using logic and mathematics

Historical Example: The St. Petersburg Paradox

- Game:
  - You get to toss a fair coin for as many times as you need to score a “head” (H)
  - n is the toss on which the first H appears: 1 <= n <= infinity
- Payoff:
  - You get $\$2^{n}$
    - If you score H on toss 1, you get $2
    - If you score H on toss 2, you get $4
    - If you score H on toss 3, you get $8
    - If you score H on toss 4, you get $16, etc.
- Question:
  - How much are you willing to pay me in order to play this game for one round?
  - How do you decide???

Expected Value of one Round of the Game

- How much do you think you can expect to win in one round of this game?

- $EV(X) = \Sigma_i (x_i \; p(x_i)) = ?$

- Is EV a good decision rule for how much to pay for this game?
  - No!
  - Instead, Bernoulli (1834) suggested that we compute expected utility of outcomes, where utility is decreasing over amount
    - Logarithmic function

- Expected Utility of Game
  - Daniel Bernoulli (1739)
    - Utility of wealth is not linear, but logarithmic
    - $EU(X) = \Sigma_i \{u(x_i)\ p(x_i)\}$
- Other decision rules
    - **Minimum return (pessimist) rule:**
      - pay no more than you can expect to get back in the worst case
    - **Expectation heuristic (Treisman, 1986):**
      - figure on what trial you can expect to get the first H and pay no more than you will get on that trial

Examples where EV is a good decision rule

- Pricing insurance premiums
  - Actuaries are experts at getting the relevant information that goes into calculating the expected value of a particular policy
- Testing whether slot machines follow state laws about required payout

Expected Utility Theory

- Generally considered best normative “objective function” since its axiomatization by von Neumann & Morgenstern (1947)
  - Rationality axioms seem reasonable and desirable
  - EU maximization follows (deductively) from axioms and does not depend on any “long-run” argument

Expected-Utility Axioms
(Von Neumann & Morgenstern,1947)

- Connectedness

  x>=y or y>=x
- Transitivity

  If x>=y and y>=z, then x>=z
- Substitution Axiom or Sure-thing principle

  If x>=y, then (x,p,z) >= (y,p,z) for all p and z

- If you "buy into" all axioms, then you will choose X over Y
  - if and only if EU(X) > EU(Y),

    where $EU(X) = \Sigma_i \{u(x_i)\, p(x_i)\}$

    and $EU(Y) = \Sigma_i \{u(y_i)\, p(y_i)\}$

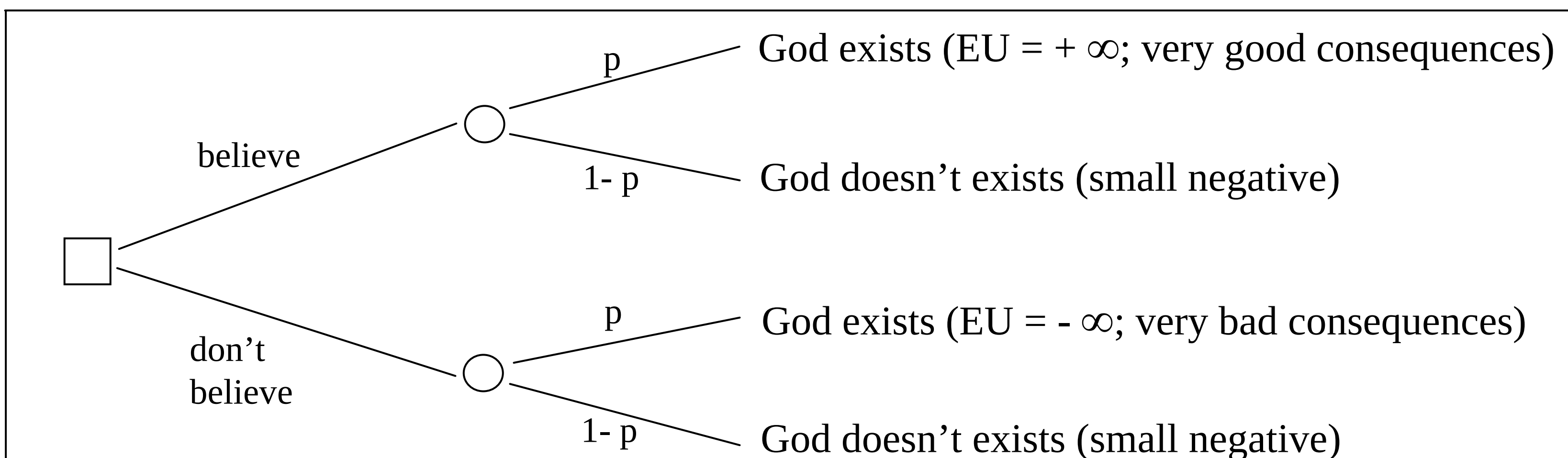
Discounted Utility Model

- For outcomes that occur not now, but later in time, utility of the outcome is discounted by a factor d
  - Discount factor d indicates how much a dollar received now would be worth if it is received in a year
    - d=1 means that there is no discounting: one dollar in a year is valued the same now as a dollar now
    - d=.50 means that there is some discouting: one dollar in a year is equivalent to receiving 50 cents now

Decision Analysis as a Way to Implement EU Maximization

- Structuring of the decision
  - Decision tree
    - Action nodes
    - Chance nodes
      - Probabilities need to be assessed
      - Utilities of component dimensions and tradeoff coefficients need to be assessed
- How to get those values?
  - Direct ways
    - Ask decision maker or experts directly
      - “how likely is given event?” (absolute judgment)
      - “how useful/valuable is given outcome?” (relative judgment)
  - Indirect ways
    - From logic or past experience
    - Ask decision maker about hypothetical decisions (“standard gambles”)
      - Work backwards from choice to determine underlying utilities

	States of nature		
	God exists	God does not exist	
	p	1- p	
Action 1: "Believe"	Utility = + ∞ (infinitely positive)	Utility = - e (very small negative)	EU("believe") = ∞
Action 2: "Don't believe"	Utility = - ∞ (infinitely negative)	Utility = + e (very small positive)	EU("don't believe) = - ∞



Therefore, according to Pascal, believing in God is a dominating alternative if you want to maximize expected utility!

What do normative/prescriptive models provide?

- Consistency in choices
- Structure for decision making process
- Transparency of reasons for choice
- Justifiability
- "Education" of other choice processes

Multi-Attribute Utility Theory (MAUT)

- Model of riskless choice
  - Choice of consumer products, restaurants, etc.
- Need to specify
  - Dimensions of choice alternatives that enter into decision
  - Value of each alternative on those dimensions
  - Importance weights of dimensions given ranges (acceptable tradeoff)
- Tradeoffs
  - Willingness to interchange *x* units of Dimension1 for *y* units of Dimension 2
  - Computer programs can help you with utility assessment and tradeoff assessment

Renting Land Example

- MAU(Rental) = b_w u(Price/ht) + b_s u(Payment Option) + b_l u(Soil Quality)
  - u(.) are the utility functions on individual rental attributes
  - b's are the importance weights of attributes
- Possible Interactions

Utility Elicitation Method

- “standard gamble” methods
  - using certainty equivalents
    - Compare lottery against sure thing of equal EV and adjust sure thing value until two options equally valued
  - use probability equivalents
    - Compare two lotteries and adjust one probability level until two options equally valued
  - why would elicitation method make a difference?
    - “Stay tuned” for prospect theory and certainty effect